# Projections

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### Introduction

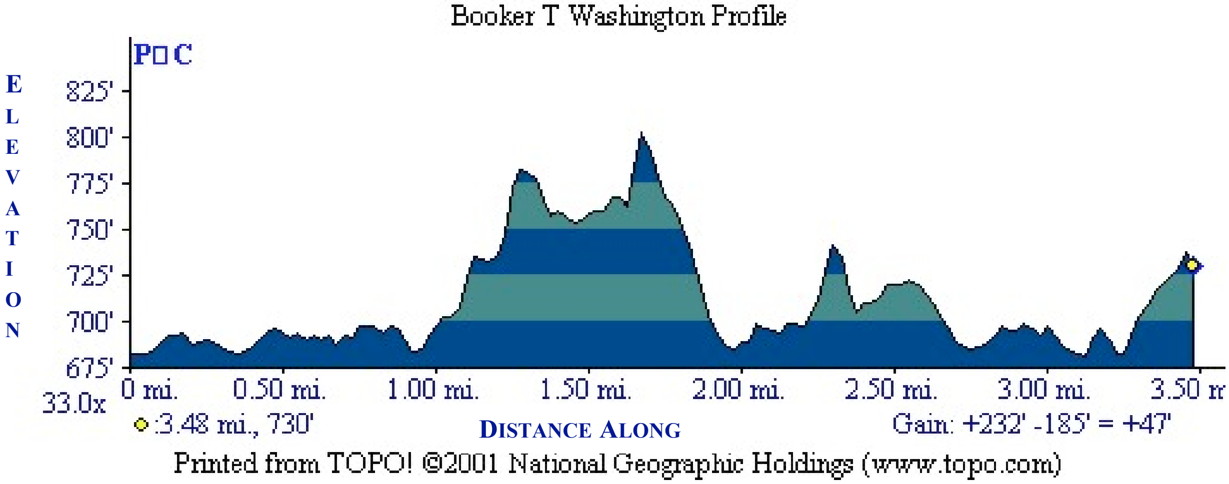
Maps and graphs are nearly always shown in 2 dimensions. This choice of presentation isn't surprising, since we look at maps and graphs on paper and computer screens. But in a lot of cases we want to look at more than 2 **variables**. Many things that we are interested in are complicated enough that we need more than 2 numbers to classify them. After all, we live in a Universe in which we experience 4 dimensions (left-right, forward-backward, up-down, and before-after). To describe an athlete's performance, statisticians need a lot more than 2 numbers. You could map all the players in the NBA onto a chart where one axis was points per game and another was rebounds per game, but then what about steals and assists, or minutes played? The question, then, is how to include as much data as possible into a format that is easy to understand? After all, the purpose of mapping is to make something understandable. A map of an area is a lot easier to read and interpret and takes up much less space than a verbal description of where every street, building, and tree is located.

In this lab we will investigate how to include more data in a map. Imagine that you're traveling from one place to another. If you know the latitude and longitude of your start point and end points, you can look at a map and find a direct path. A map will probably show you if there are any large bodies of water that you will have to go around, and likely which roads go in the right direction. But there are many other things that you might want to know about the places you will have to traverse. Will you have to pass through any places that are very hot or cold? Any places with a lot of rainfall, or any deserts? One of the most basic and frequently represented variables on maps is **elevation**. We represent elevation on maps because sometimes it is easier to go around a hill or mountain rather than over it, even if going over would be the straightest path.

## Part 1 - Tracking Elevation Changes

Let's say you have a given path. It is nice to know where along the trip any steep climbs or drops are. These types of graphs are used in long-distance bicycle races like the Tour de France, or as hiking guides. So let's make our own graph. The independent variable (x-axis) will be the distance from the starting point, and the dependent variable (y-axis) will be the elevation above the starting point. Since we are traveling along a path, let's represent our trip as a line that starts at the starting point (by definition, 0 on the x-axis and 0 on the y-axis). Now we'll need to measure how much the path goes up or down as we travel along it.

For example, here's a graph of elevation along a trail in Booker T. Washington State Park, in Chattanooga, Tennessee:




**Materials**:

- Scope level
- Meter stick
- String with meters marked on it
- Hanging level
- Plumb line

**Procedure**:

Use these tools to measure the distance and elevation gain for 6-10 steps along your path:

1. Have one person stand a short distance uphill of two partners.
2. One downhill person should hold the meter stick to the ground with the plumb line, making sure the meter stick is perfectly vertical.
3. The other downhill person should look through the scope and direct the uphill person to find a point exactly level with the top of the meter stick. Mark this uphill spot.
4. Using the string, measure the horizontal distance between the two points.
5. You now have horizontal and vertical coordinates for a point. Repeat the process, going farther uphill.
6. Now plot the points on your graph. Remember that the elevation gain (or loss) of each step is measured from the previous step, not from the starting point.

**In your lab notebook, answer the following**:

1. What does your graph tell you?
2. What information is missing from your graph?
3. In what situations is this graph useful, and in what situations would it not be useful?
4. What are the major sources of uncertainty/error in your graph?

## Why Use Contour Maps?

The chart we made of elevation vs. distance along a path can be useful, but only in limited circumstances. What if someone wants to choose their own path through a region? What if someone wants to know what they will be passing not only directly on their path, but also nearby? Or what if the person just wants to know about the direction and twists and turns of their path as well as the elevation gain and loss?

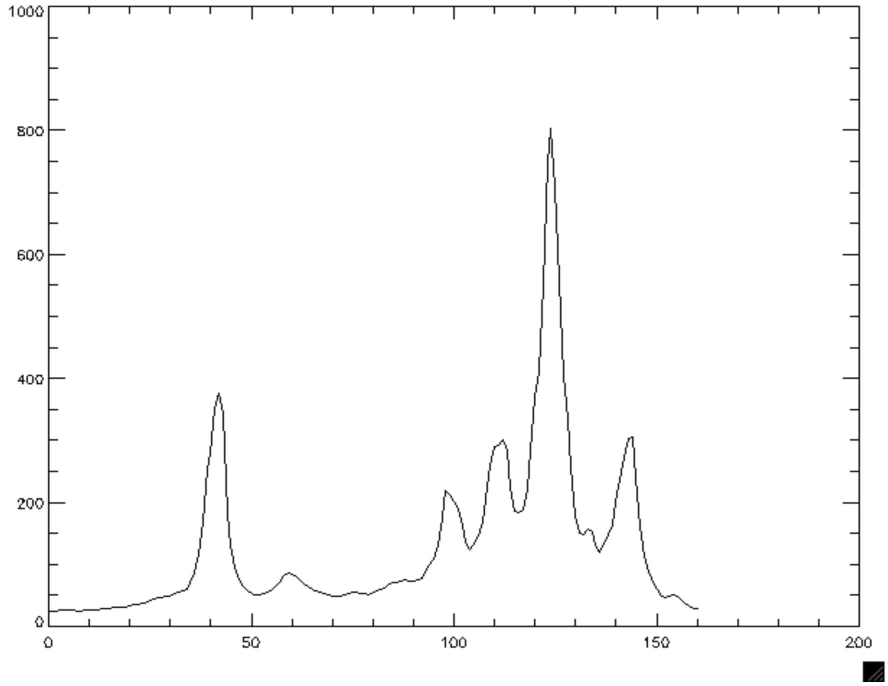
As we've seen throughout the Institute this year, one of the best ways to get a lot of information into an easily readable form is to make a map with a two-dimensional coordinate system and features plotted at their respective locations. In many cases, in addition to (or instead of) features like roads, trees, and buildings, we may also want to plot a third variable, such as temperature, air pressure, rainfall, or elevation. How do we plot this third variable? One way is with color. You've probably seen weather maps where the places that are hotter appear in red, and the places that are colder appear in blue. Astronomers often use a similar technique to depict areas of the sky from which more or less light reaches us on earth. Color is easy for us to see and understand quickly, but it is hard to quantify. For example, on a map of a galaxy, how could you tell which color corresponds to twice the brightness of another color?

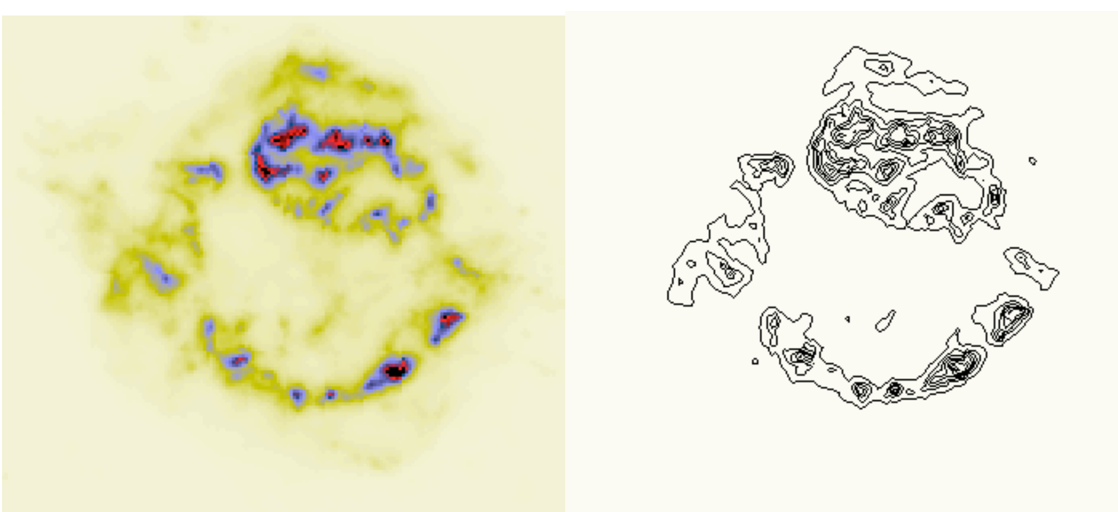
Below is an example of a weather map where color designates rainfall. It's printed in black and white, but you can still see the difference in shades.


Image source: weather.com

A more quantitative method is to use **contours** to depict different levels. Contours are lines drawn on a map where every point on the line has the same value for the variable being plotted. For instance, on a temperature map of the United States, there might be a contour where every place that the line crosses is at a temperature of 70 degrees. This is useful because to find the value of the variable at any point, you just have to see which contour crosses that point.

Below are three representations of the same object. The image is of the supernova remnant Cassiopeia A, taken by the Spitzer Space telescope.



The top image is a graph of brightness vs. position in the sky, where the position is taken in a vertical slice through the middle of the picture. The bottom two are the same picture, with brightness at each position on the sky represented by shade of gray in one case (left) and contours in the other (right).

## Part 2 - Making a Contour Map

In this part of the lab, we'll use the technique of contour mapping to depict the elevation across a certain landscape.

Here's the trick we'll use: water settles to be a flat surface, so if we add a certain amount of water to a landscape, all the points along the shoreline of that water will have the same elevation. If we draw lines along the shoreline, each line will form a path of constant elevation, which is the definition of an elevation contour.

**Materials:**

- Fish tank with landscape
- Water
- Ruler/tape measure
- Plumb line

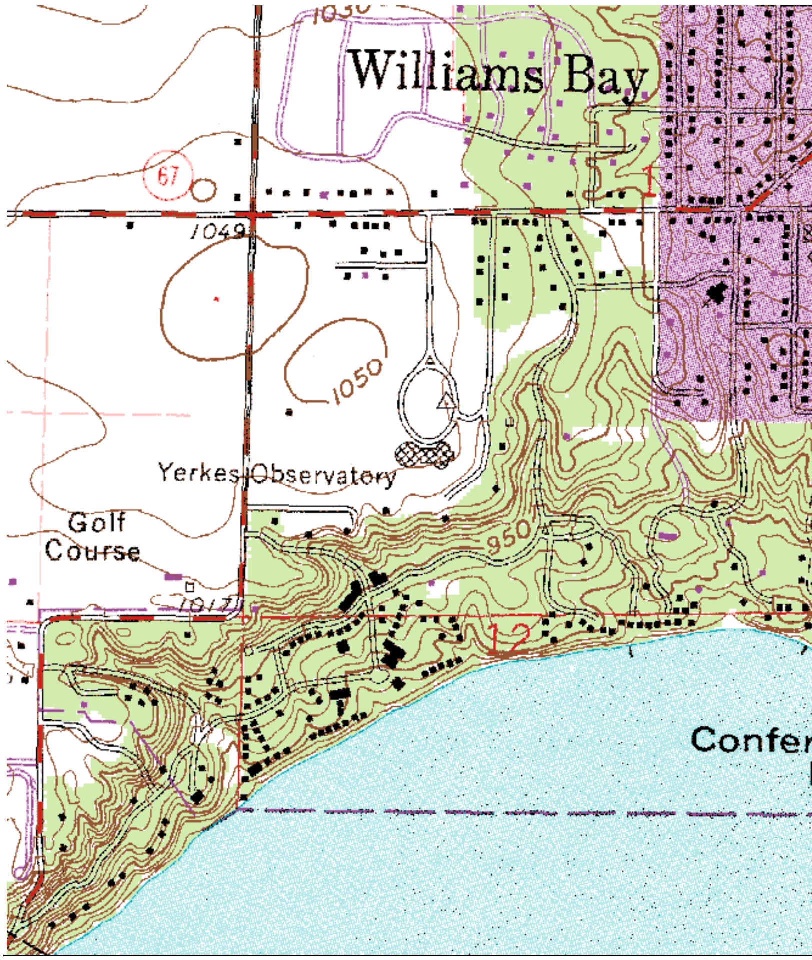
**Procedure**:

1. First, decide on a scale to use for your map. You want the map to be as big as possible, but still fit on the paper you are using. Write down the scale you are using.
2. Make your axes according to the scale you have chosen to use.
3. Measuring up the side of the fish tank, mark points every half-inch.
4. Pour water into your tank until the water level is even with your *lowest* mark.
5. Measure the shape and position of the "shoreline" and replicate it on your map. One way to do this is:
    (a) Find and mark any key points along the shoreline, like sharp corners and points where the shoreline meets the tank wall.
    (b) Resting a ruler or tape measure across the top of the tank, with the plumb line measure the distance of the key points from the sides of the tank.
    (c) Draw those points on your map, and estimate the smooth lines between them.
6. Pour more water in until the water level is even with your next mark.
7. Repeat steps 4 and 5 for each successively higher mark on the side of the fish tank until the land is completely covered with water.

**In your lab notebook, answer the following:**

1. What does the landscape you made a map of look like? Describe it briefly in words.
2. What are the most important features of the landscape? Are these important features visible on your contour map?
3. Points A and B are labeled on your landscape. Pick out two routes between them, one that avoids steep climbs and drops, and another that includes them. Now sketch these routes onto your contour map. What do you notice about the routes and the contour lines?

4. As we noted, the shores of a lake or ocean define a contour, so a contour map of a region with a lake or ocean would have one contour running along the shore. Is this true for a river? In other words, will a single contour run along the banks of a river?
5. Can you think of a different variable than elevation that we might make contours of on a map?




Contour Map of area around Yerkes Observatory.
Source: USGS

## BIG QUESTIONS:

1. You are designing the next mission to Mars. Given a contour map of the planet, how would you decide where to land your spacecraft and why?

2. Write about how you might figure out roughly how elevation changes by looking at a map without contours on it. What clues to the placement of mountains can you pick up from, say, a satellite image?

3. What methods do you think Lewis and Clark could have used to make maps when they tried to map out the northwestern United States? What methods do you think Geological Surveys use now to make contour maps?